

3D Matching Problem is NP-complete

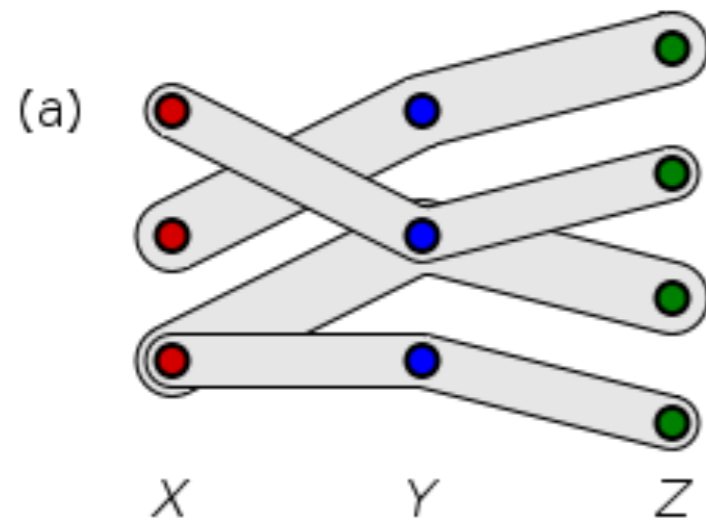
3D Matching Problem is **NP-complete**

- Let X , Y , and Z be finite, *disjoint sets*.
- Let T be a subset of $X \times Y \times Z$
- T consists of triples (x, y, z) such that $x \in X$, $y \in Y$, and $z \in Z$

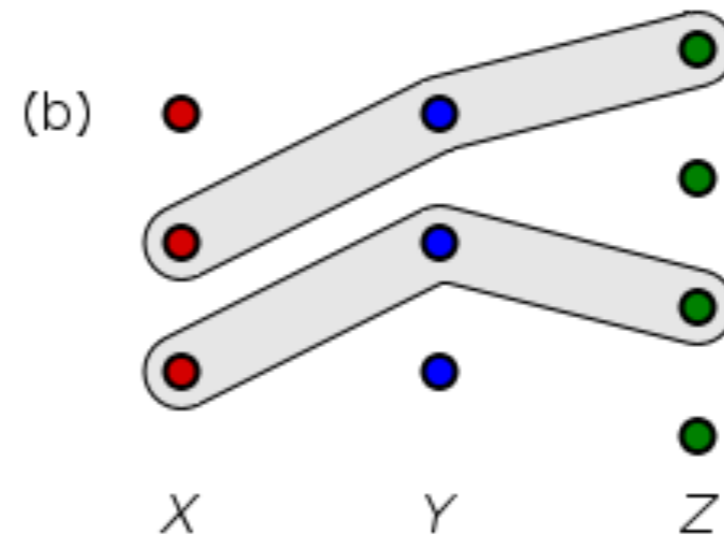
$M \subseteq T$ is a 3DM if:

Any two distinct triples $(x_1, y_1, z_1) \in M$ and $(x_2, y_2, z_2) \in M$,
where $x_1 \neq x_2$, $y_1 \neq y_2$, and $z_1 \neq z_2$

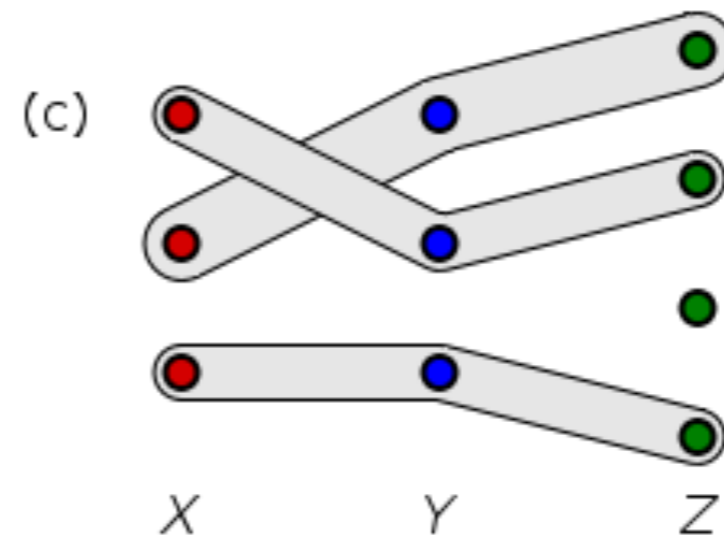
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T



M1



M2

3D Matching Problem is **NP-complete**

Definition.

Given disjoint sets X , Y , and Z , each of n elements, triples $T \subseteq X \times Y \times Z$.

Is there a subset $M \subseteq T$ such that each element $\in X \cup Y \cup Z$ is in exactly one $m \in M$?

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PART I: 3DM belongs to NP.

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PART I: 3DM belongs to NP.

Given a *certificate*, which lists a candidate list of triples, a *verifier* can check that each triple belongs to T and every element of $X \cup Y \cup Z$ is in one triple.

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PART II: 3DM is NP-hard.

A reduction from **3SAT**. By showing that $3SAT \leq_p 3DM$

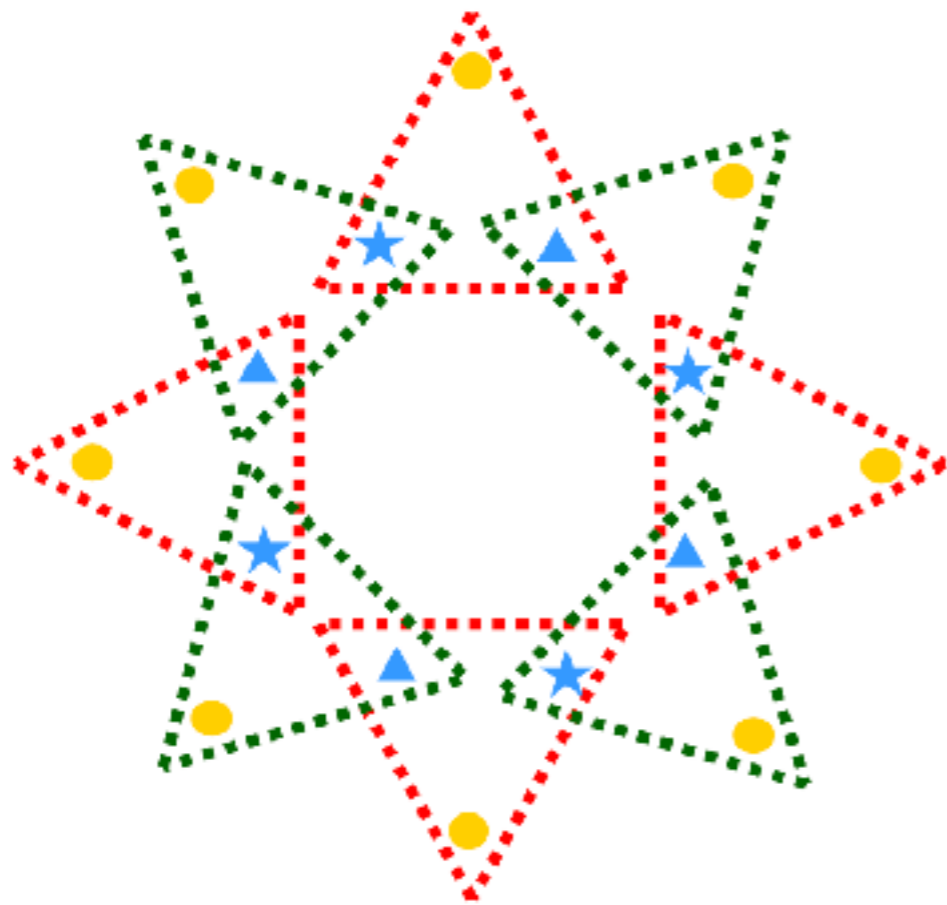
We build gadgets for the variable and clauses.

Given: A 3SAT instance with k clauses and n variables.

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PART II: $3SAT \leq_p 3DM$ | Variable

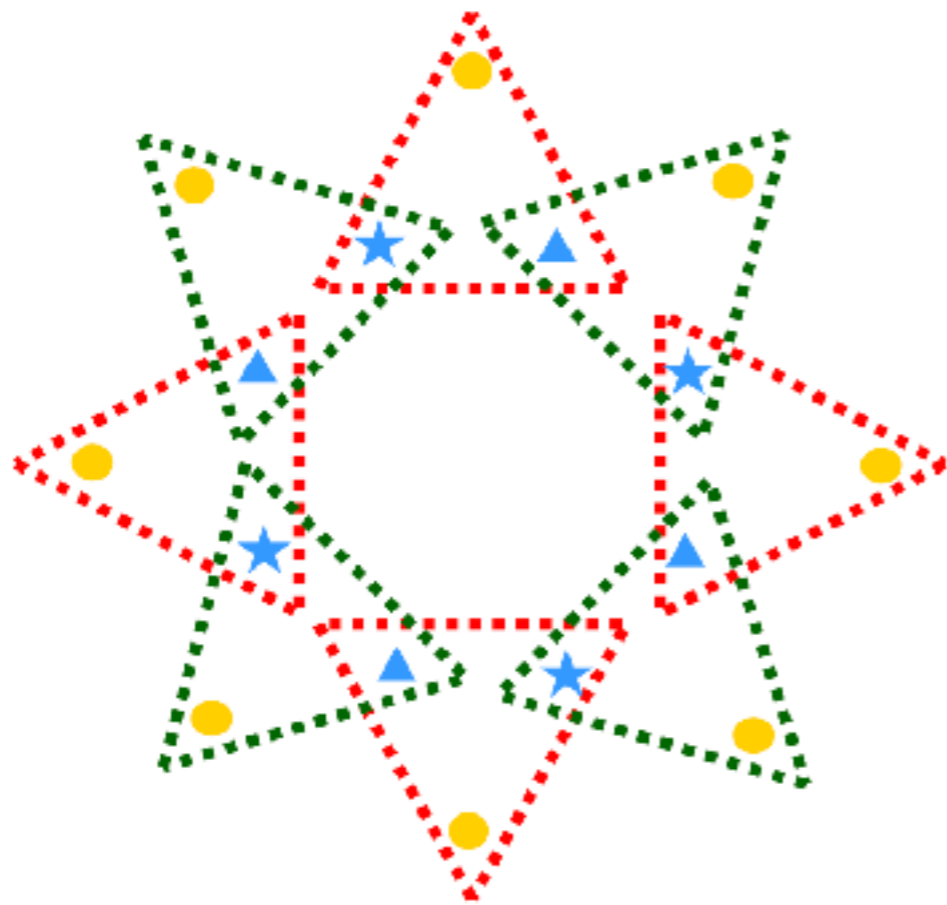
Let variable x_i occurs r_i times, which create r_i red and r_i green triangles linked in a circle, one pair per occurrence.



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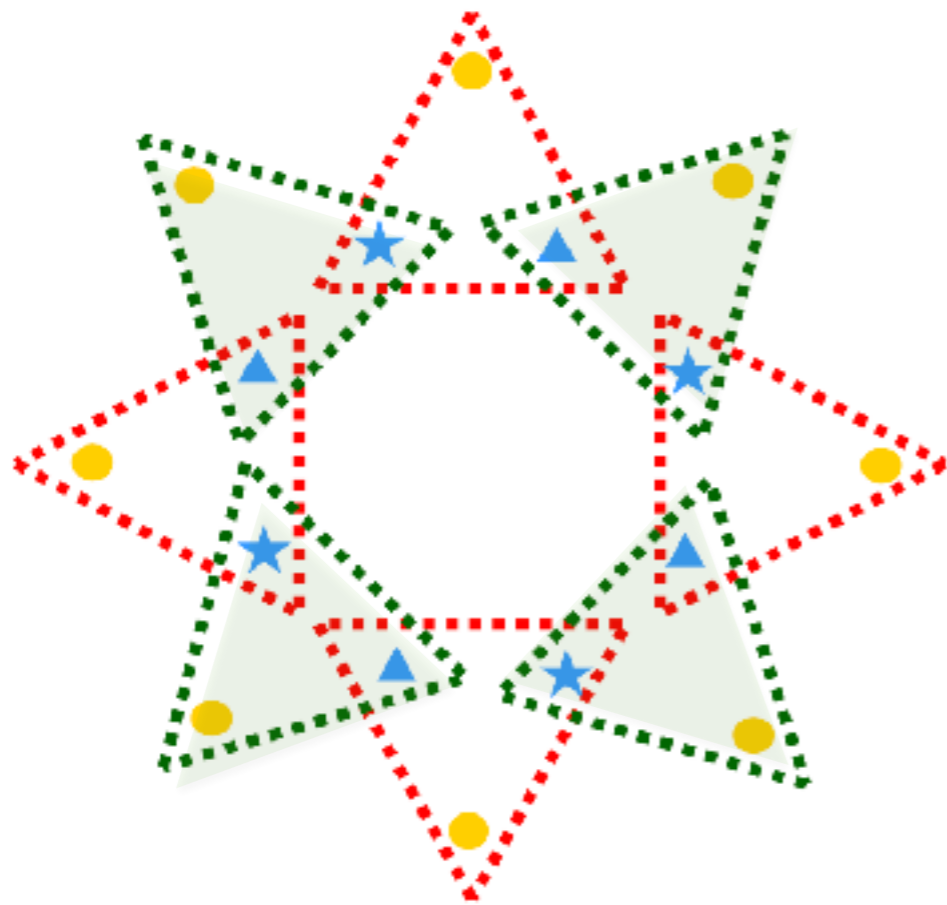
Perfect matching M must;

- Use the **green** edges, leaving **red** tips uncovered if x_i is assigned **false**.
- Use the **red** edges, leaving **green** tips uncovered if x_i is assigned **true**.

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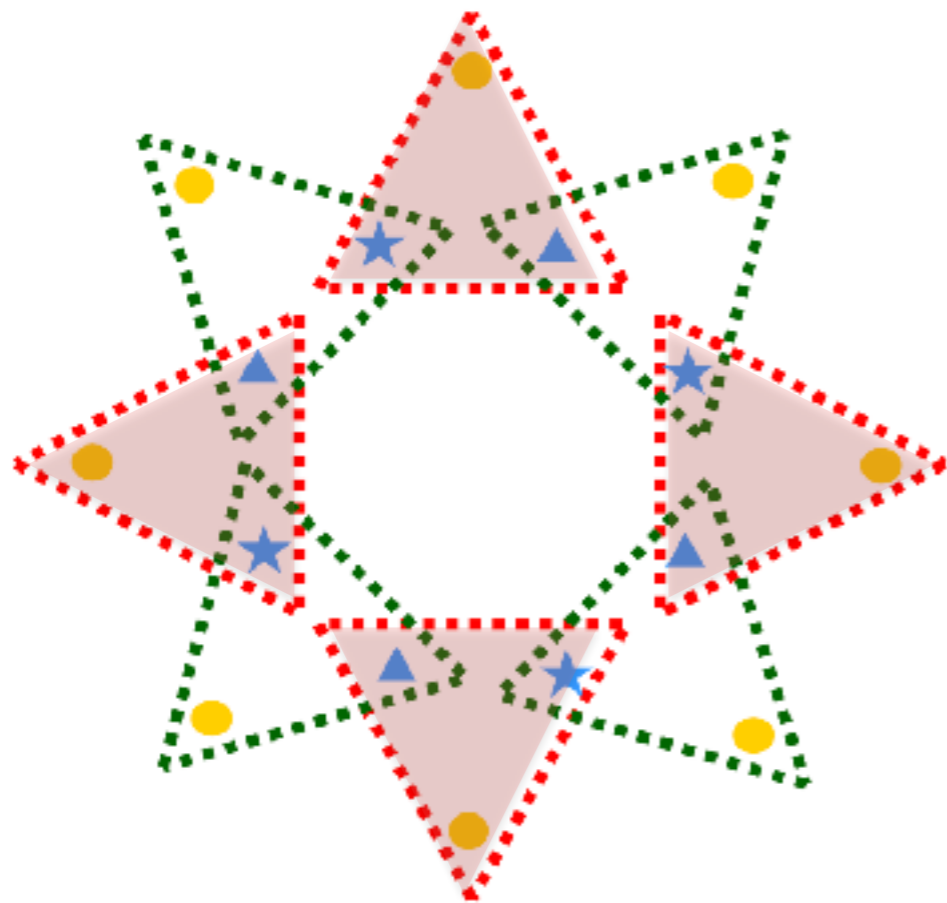
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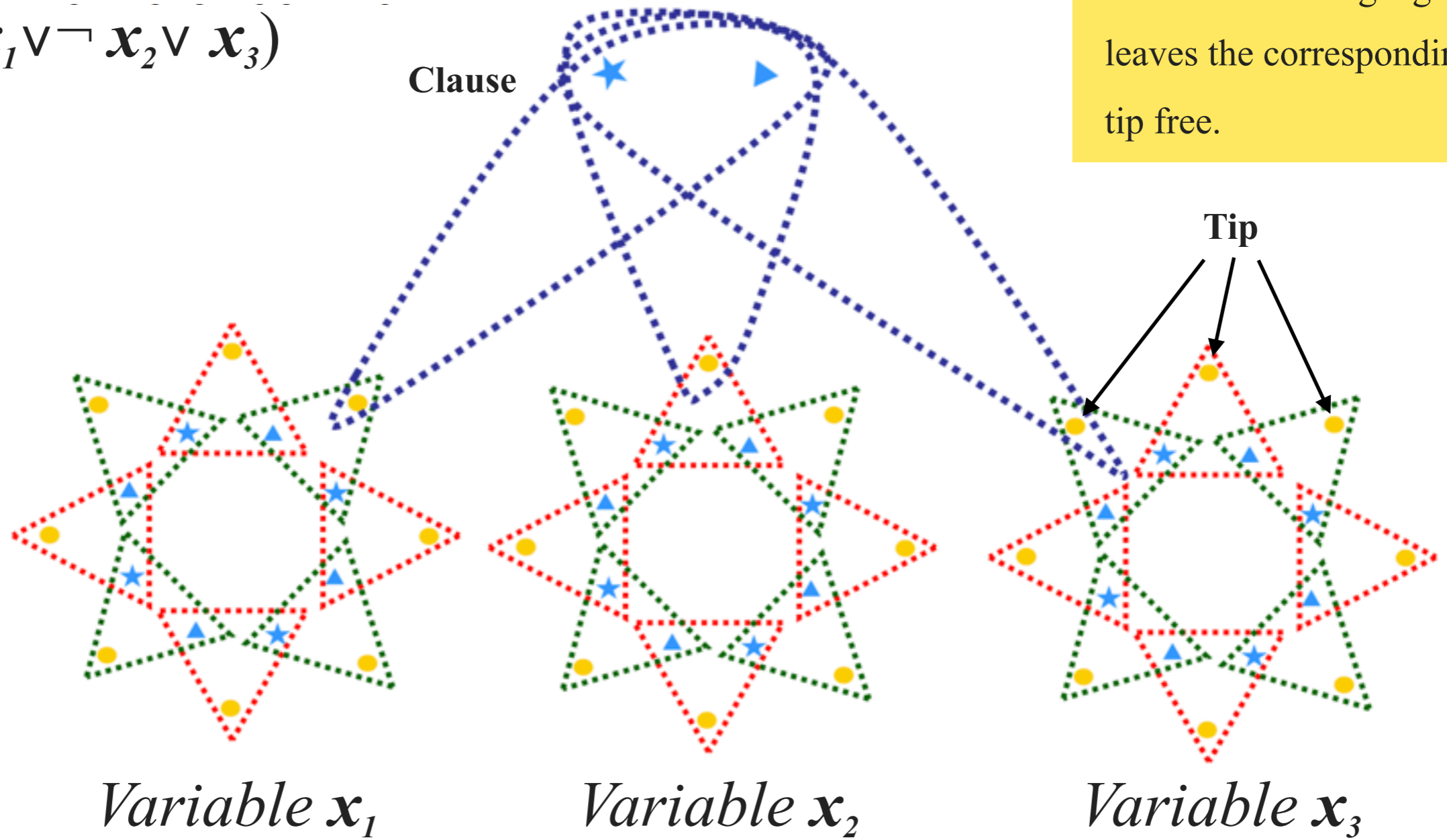
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PART II: $3SAT \leq_p 3DM$ | Clause

$$C_1 = (x_1 \vee \neg x_2 \vee x_3)$$



Clause matched

If some variable gadget leaves the corresponding tip free.

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PART II: $3SAT \leq_p 3DM$ | Clauses

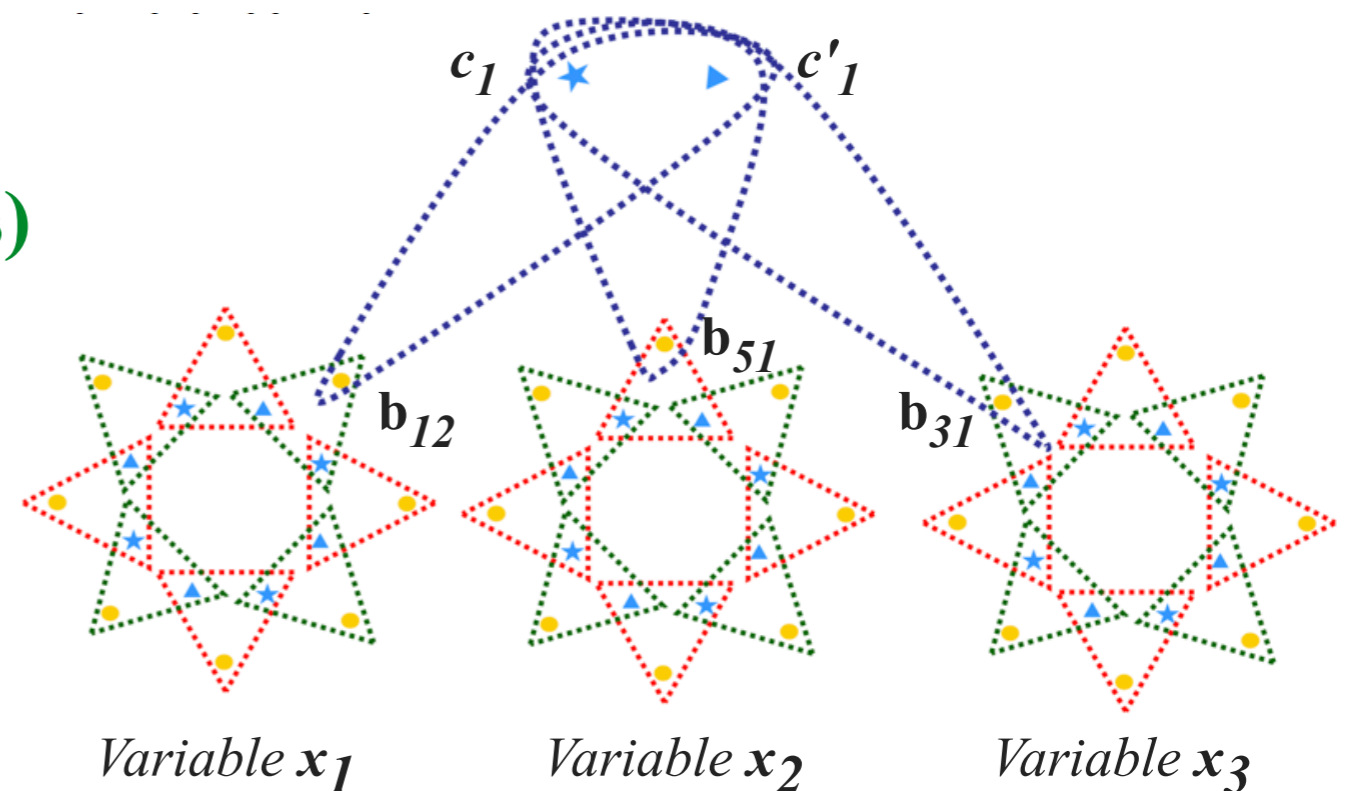
We create a gadget for each clause $k = \{t_1, t_2, t_3\}$

Clause k for clause c_k contains two core elements $\{c_k, c'_k\}$ with

three triple: (c_k, c'_k, b_{i2k}) for c_k contains x_i

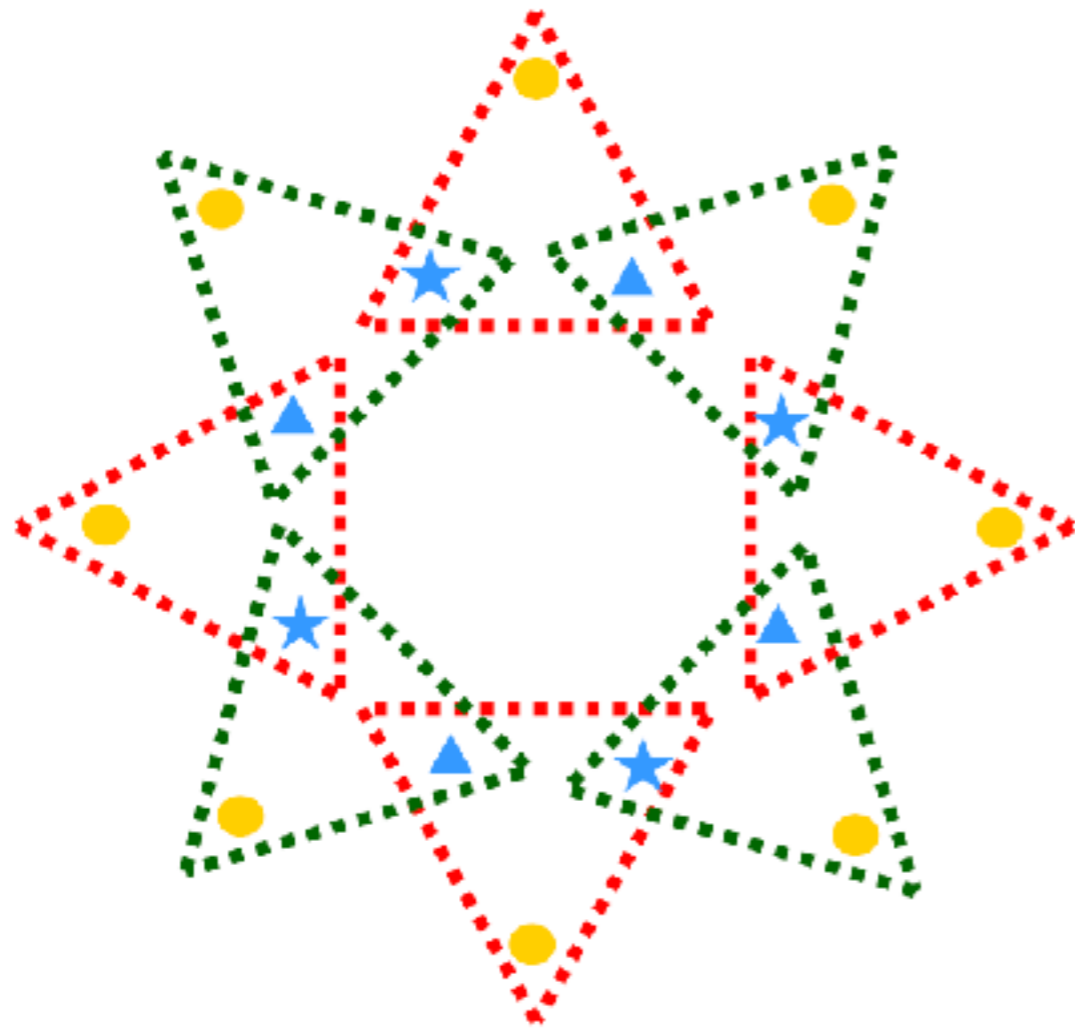
(c_k, c'_k, b_{i2k-1}) for c'_k contains $\neg x_i$

We hook up the clauses (core nodes)
with some tip nodes depending
the variable to be true or false.



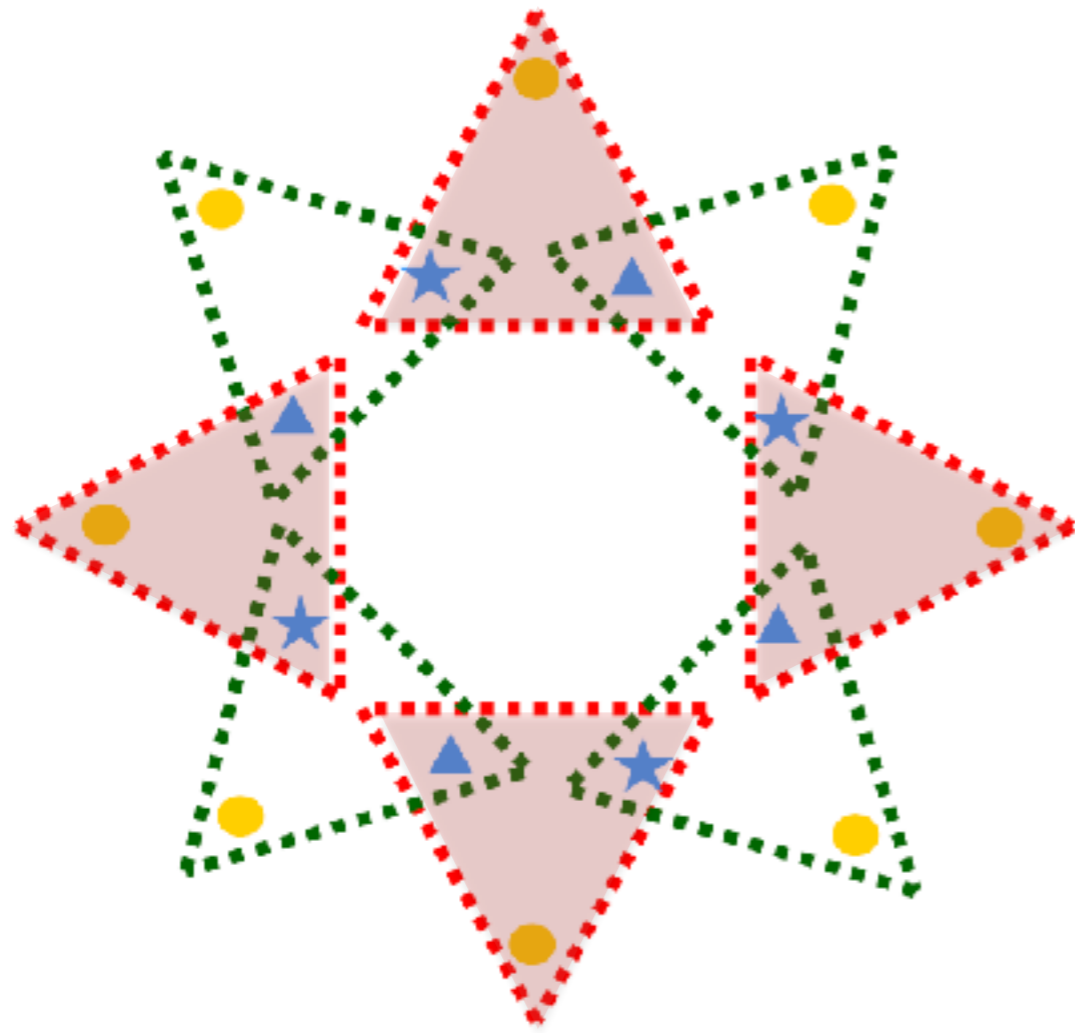
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PART II: $3SAT \leq_p 3DM$ | Slack



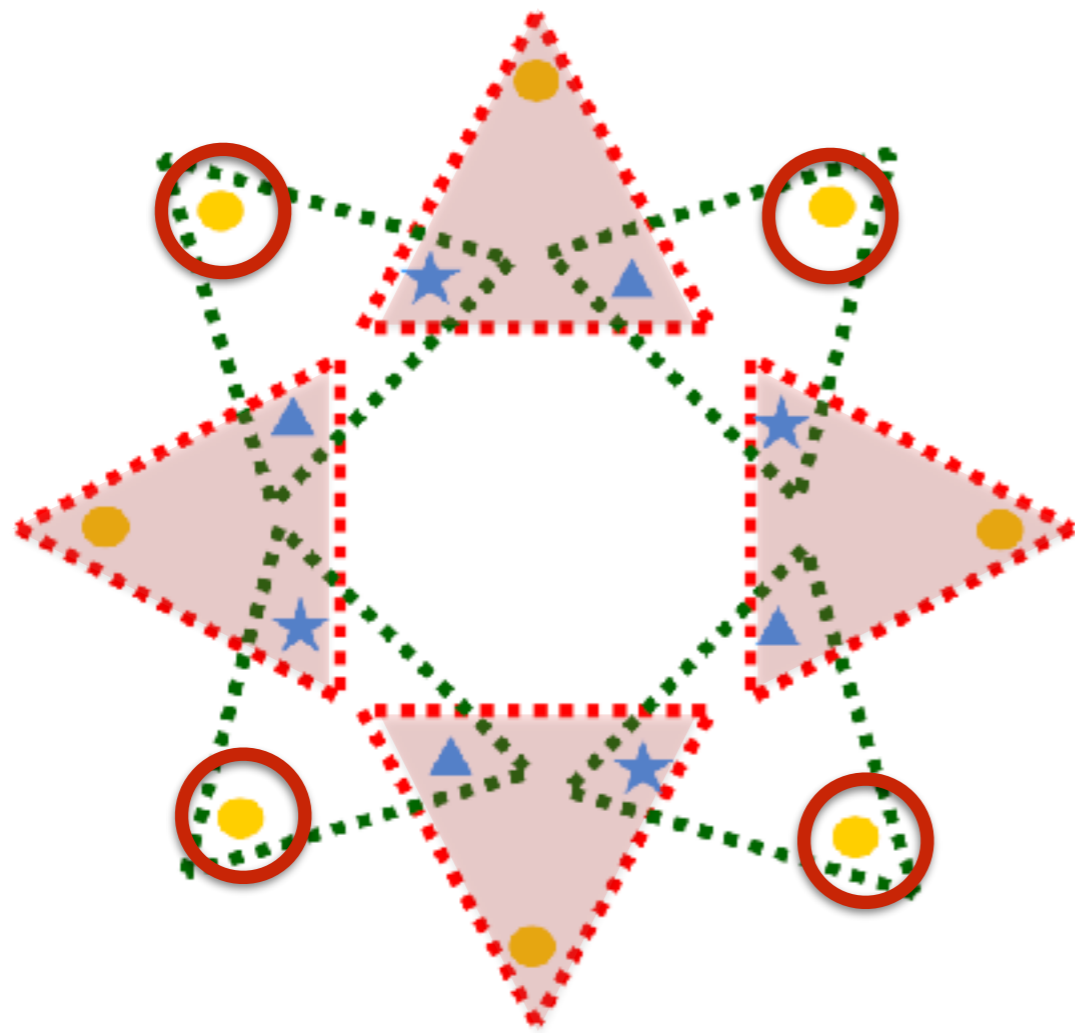
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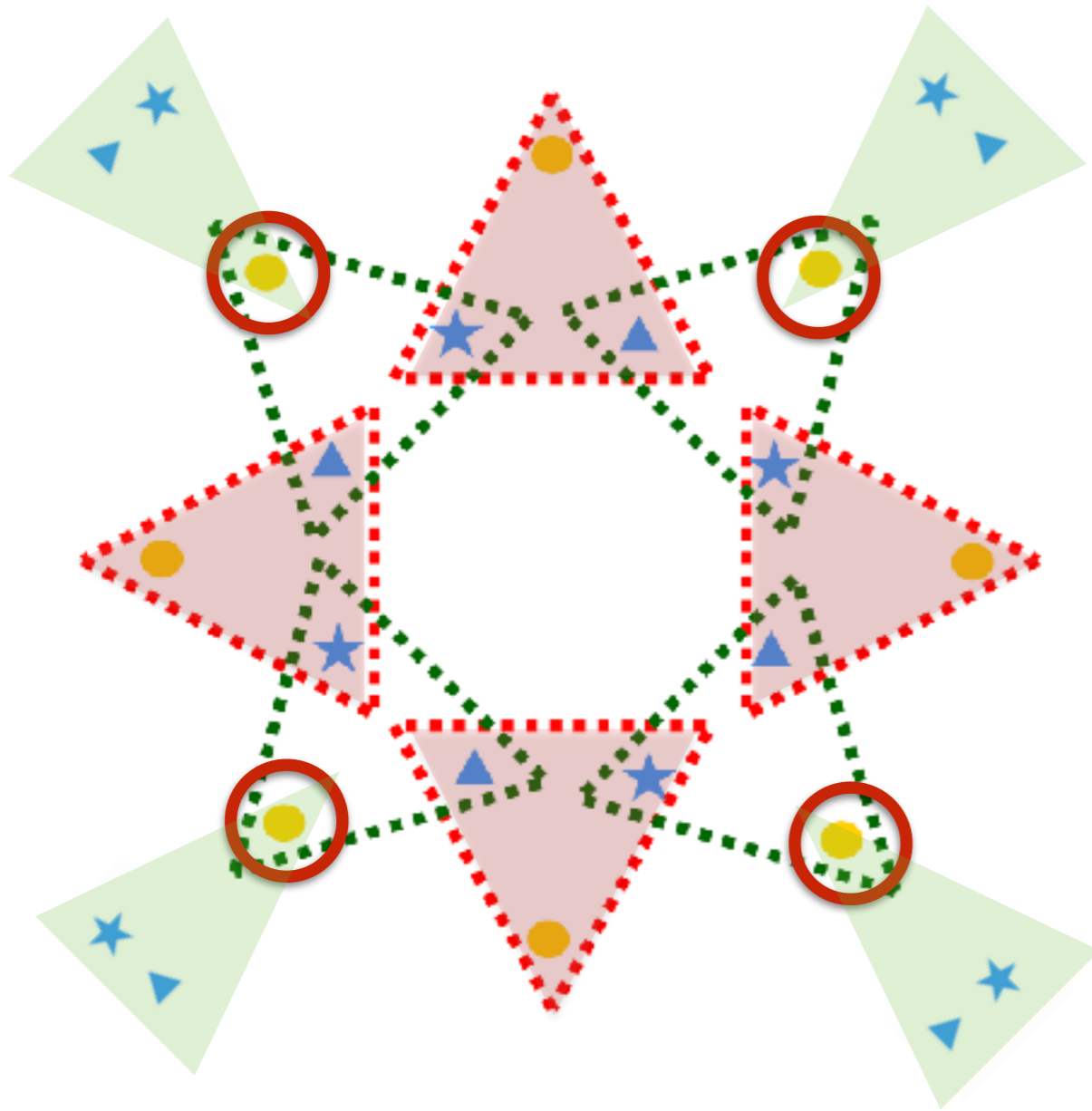
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