Solving Werewolf Problem

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problem from http://acm.timus.ru/problem.aspx?num=1242

Werewolf problem

- We have villagers in a village
- Most of them are each other's relative
- Werewolves are among those villagers
- Werewolves are never kill their ancestors and descendants

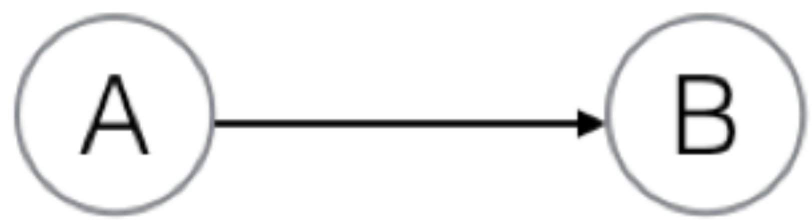
Werewolf problem (2)

- The problem provides
  - The set of death villagers killed by werewolves
  - The number of villagers in the village
  - All relationships between villagers

- The problem wants the set of villager who suspect to be werewolf

Problem Modelling

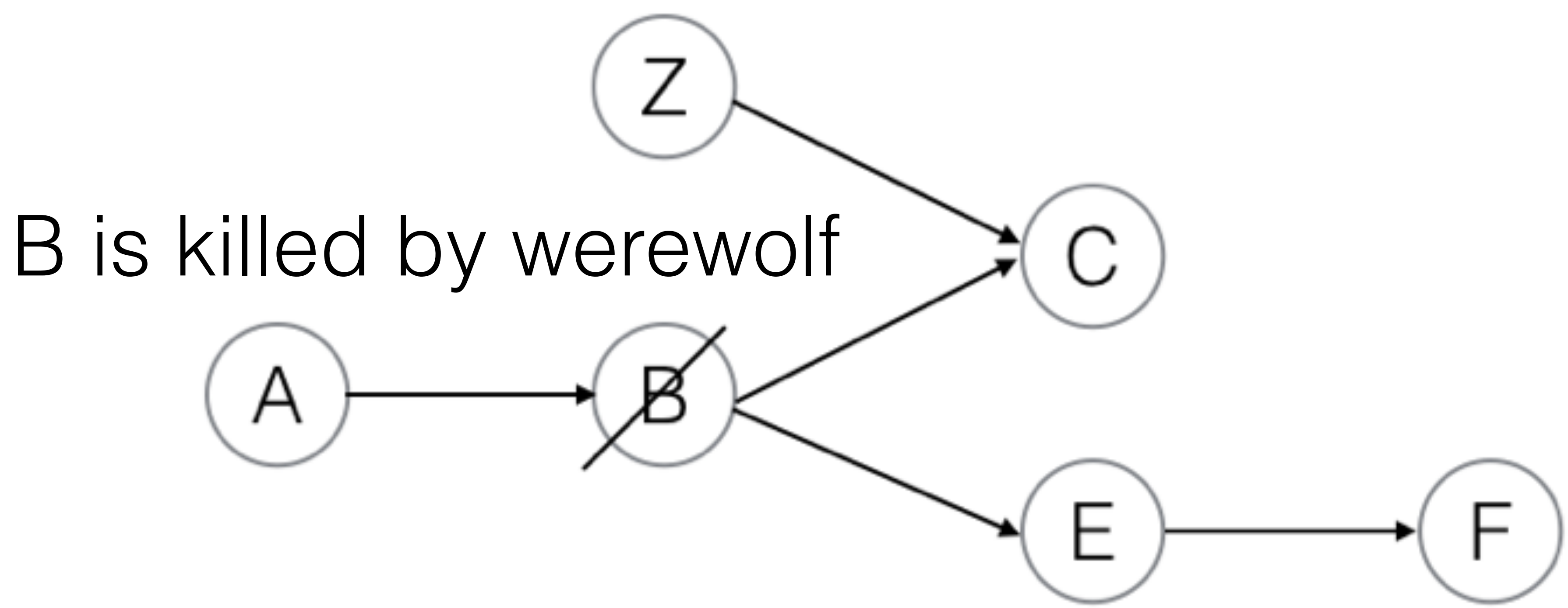
- Use graph to represent the entire village



2 villagers (A, B) while A
is ancestors of B

Problem condition

- Werewolves never kill their ancestors and descendants



Z is suspect to be werewolf because Z is not ancestor or descendant of B

Idea to solve problem

- Find out all the ancestors and descendants of death villagers
- Other villagers not in the list of above are considered to be werewolf

Graph representation for werewolf problem

- Use modified **Adjacency List** representation for node

```
class Villager(object):
    ancestors = list()
    descendants = list()
```

- Easier to directly locate all ancestors and descendants when the node (villager) is known

The algorithm

```
WERWOLF(N)
  Q = ϕ
  for each death s ∈ N
    s.visited_ancestor = true
    ENQUEUE(Q, s)
  while Q ≠ ϕ
    u = DEQUEUE(Q)
    for each v ∈ u.ancestors
      if v.visited_ancestor = false
        v.visited_ancestor = true
        ENQUEUE(Q, v)

for each death s ∈ N
  s.visited_descendant = true
  ENQUEUE(Q, s)
while Q ≠ ϕ
  u = DEQUEUE(Q)
  for each v ∈ u.descendants
    if v.visited_descendant = false
      v.visited_descendant = true
      ENQUEUE(Q, v)

for each s ∈ N
  if v.visited_ancestor = false and v.visited_descendant = false
    ENQUEUE(Q, s)

Q is now contains the suspect villagers
```

The algorithm (2)

- It is a modified BFS (breath-first-search) for graph traversal

- Doing traversal 2 times
  - one for ancestor
  - another for descendant

- All nodes (villager) not visited by those 2 traversals are werewolf

Running time analysis

- Loop though all death villagers can be at most the number of all villagers- O(V)
- While loop only run for once for each villager because of the ancestor flag - O(V)
- Ancestor list is iterate once for each villager, at most equal to number of all edges - O(E)
- Do the same thing for descendant part - All above multiply by 2
- Lastly loop through all the villagers - O(V)
- O(2V + 2V + 2E + V) -> O(V + E)

Proof of correctness

- **Claim 1** - All normal villager need to be visited at least once

  ENQUEUE(*Q*, *v*) will add node to be visited when the node is either ancestor or descendant of death villager by the flag ***visited_ancestor*** and ***visited_descendant***

- **Claim 2** - All ancestors and descendants of death villager can be reached from death villager

  From claim 1 villager will be visited by either from ancestor or descendant relationship or both if the algorithm cannot find villager anymore to add to ***Q*** that mean all ancestors or descendants are already found because from claim 1 the villager will not repeat itself in each ancestor or descendant part which is the result of flag (***visited_ancestor, visited_descendant)***